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Accretion-driven core collapse and the collisional formation of massive stars

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ABSTRACT

We consider the conditions required for a cluster core to shrink, by adiabatic accretion of gas from the surrounding cluster, to densities such that stellar collisions are a likely outcome. We show that the maximum densities attained, and hence the viability of collisions, depends on a competition between core shrinkage (driven by accretion) and core puffing up (driven by relaxation effects). The expected number of collisions scales as $N_{core}^{5/3}\tilde{v}^2$ where N_{core} is the number of stars in the cluster core and $\tilde{v}$ is the free fall velocity of the parent cluster (gas reservoir). Thus whereas collisions are very unlikely in a relatively low mass, low internal velocity system such as the Orion Nebula Cluster, they become considerably more important at the mass and velocity scale characteristic of globular clusters. Thus stellar collisions in response to accretion induced core shrinkage remains a viable prospect in more massive clusters, and may contribute to the production of intermediate mass black holes in these systems.

Key words: celestial mechanics, stellar dynamics - stars:formation - galaxies:star clusters

1 INTRODUCTION

There has been considerable speculation in recent years that the cores of clusters may be particularly conducive to the formation of very massive stars, and their remnants. It is well known that the most massive stars in young clusters tend to be concentrated towards the centre (e.g. Hillenbrand & Hartmann 1998, Sirianni et al 2002, Chen et al 2007) and that, at least in some cases, this mass segregation must be primordial (i.e. not purely the effect of two body relaxation in a cluster with an initially uniform stellar mass spectrum: see Bonnell & Davies 1998, McMillan et al 2007 for investigations of this issue). There is also some observational evidence for massive ($< 10^3 - 10^4 M_{\odot}$) remnants in some old globular clusters (Gerssen et al 2003, van der Marel 2004, Noyola et al 2006), which would be consistent with these objects following the same relationship between black hole mass and host system mass as is found in galactic bulges. Such intermediate mass black holes in globular clusters could either represent the evolutionary end-products of very massive stars in the cluster core or else could have been formed through the merger of a number of black holes of more modest mass (i.e. in the normal stellar regime), themselves being the end-products of stellar evolution of massive stars in the cluster core (van der Marel 2004, Gürkan et al 2004). In either case, it is important to know what is the mass spectrum of massive stars in the cluster core – whether it can be described purely by a normal (Salpeter) IMF modified by dynamical mass segregation effects, or whether there are other processes that enhance the production of massive stars in the cores of dense star clusters.

One possibility is that the cores of clusters pass through a phase of such exceedingly high density ($\sim 10^8$ pc^{-3}), that there is an episode of runaway stellar collisions which may - provided, that the velocity dispersion in the cluster core remains low enough (< 500 km s^{-1}) - lead to the production of a very massive star. The densities required for stellar collisions far exceed the highest stellar densities ever observed, suggesting that any putative ultra-dense phase is short-lived; the associated mass density also far exceeds that of the densest observed gas in star forming clouds, implying that such a high density could *not* be primordial. It is thus necessary to postulate a dynamical mechanism that can drive the cores of young star clusters to the requisite high densities.

Two mechanisms have been proposed for this. The first is a purely stellar dynamical effect associated with a mass segregation instability (Spitzer 1969)in a cluster containing a range of stellar masses (Rasio et al 2003, Gürkan et al 2004). The second is associated with an earlier evolutionary stage, when the cluster is gas rich, and relies on the adiabatic shrinkage of a cluster core that is accreting gas inflowing from larger radius in the cluster (Bonnell, Bate and Zinnecker 1998).

In this Letter, we consider the issue of what limits the

stellar densities attainable in this latter case, and how this maximum density relates to the parameters of the parent cluster. We will present simple analytic arguments (which are consistent with the results of existing numerical calculations) in order to show that the central density may be driven to densities that exceed the mean cluster density by a factor that scales with the square of the mass of the parent cluster. We then go on to demonstrate that this places constraints on the parameters of clusters in which stellar collisions may become important. We find that collisions are favoured in more massive systems since these shrink further, in response to accretion, before this is reversed by two body effects.

2 THE EVOLUTION OF A CLUSTER CORE SUBJECT TO ACCRETION

2.1 Preamble

We develop some simple analytical arguments in which the system is approximated by a parent gas reservoir ('the cluster') and a dense stellar core whose density increases with time due to adiabatic accretion of gas from the surrounding cluster. (Note that the argument would apply equally well to the situation where the entire stellar content of the system resided in the core, so although we retain the above nomenclature of 'core' and 'cluster' we can also envisage a situation where, for example, the core is actually an entire cluster, located perhaps at the intersection of inflowing gaseous filaments and thus subject to a supply of accretable material: see e.g. Bonnell et al 2003).

In what follows, we omit numerical constants of order unity, since we apppreciate that our estimates are in any case in need of refinement by further numerical experiments. In particular, in this simplest analysis we treat the inner (core) system as being homogeneous (i.e. represented at any time by a single value of stellar mass, velocity dispersion and density). The utility of this analysis lies not so much in the numerical values that we derive but in its setting out of the underlying principles governing the evolution of an accreting cluster core. In particular, we draw attention to the scaling relations that we derive for how the maximum stellar density (and hence the incidence of stellar collisions) depends on the parameters of the parent cluster.

2.2 The maximum density in the core

We consider the simple situation of a cluster, mass M_{cl} with a characteristic dynamical timescale of t_{dyn}. If the cluster is largely gaseous initially, then in the absence of support mechanisms for this gas, the flow rate of gas into the central regions is

$$\dot{M} \sim \frac{M_{clus}}{t_{dyn}} \tag{1}$$

We now consider the case where star formation in the central regions produces a core of mass M_{core}, whose mass grows with time as its members intercept the accretion flow described by (1). For the purpose of this simple argument, it is assumed that all the inflowing gas is accreted by stars in the core, an assumption which is unlikely to be true in practice once stars attain high masses ($\sim 10 M_\odot$), due to the effects of feedback (e.g. Wolfire & Casinelli 1987, Yorke & Sonnhalter 2002, Edgar & Clarke 2004). A consideration of the complex effects of stellar feedback in an accreting stellar core is beyond the scope of this Letter: see e.g. Dale et al 2005 for recent simulations of this situation. Likewise, we here neglect the possibility that some of the accretion flow into the core produces new stars via gravitational instability of the inflowing gas: simulations suggest that cluster cores grow in mass by a combination of the infall of small stellar systems and the subsequent growth of these stars by further gas accretion: see Bonnell, Bate and Vine (2003) for a detailed analysis of this issue.

The timescale on which the core mass grows is thus $t_{\dot{M}}$:

$$t_{\dot{M}} \sim \frac{M_{core}}{\dot{M}} \sim \frac{M_{core}}{M_{clus}} t_{dyn} \tag{2}$$

Since this is obviously much less than the dynamical timescale of the parent cluster (t_{dyn}), we see that to first order the terms M_{clus} and t_{dyn} remain roughly constant during core growth and hence $t_{\dot{M}}$ increases linearly with M_{core}.**(We will go on to show below that the density of the core rises very steeply as the core grows, so that in fact $M_{core} << M_{clus}$ throughout the evolution we consider here. Thus the assumption that $t_{\dot{M}} << t_{dyn}$ also remains valid).** On the other hand, the *core's* dynamical time ($t_{dyn_{core}}$) decreases with time as the core density increases and hence the ratio $t_{\dot{M}}/t_{dyn_{core}}$ increases monotonically with time. Therefore, the core will at some point enter the regime where $t_{\dot{M}}/t_{dyn_{core}}$ becomes larger than unity, and, as this condition becomes more amply fulfilled with time thereafter, the core growth becomes, to a better and better approximation, adiabatic. In this case, the addition of mass on a timescale much longer than the local dynamical time, results in growth which preserves the adiabatic invariant $M_{core}v_{core}R_{core}$, where v_{core} and R_{core} are respectively the velocity dispersion and characteristic radius of the cluster core. Note that in this regime, individual stellar orbits preserve their angular momenta. Evidently, our expressions are only valid in the limit that the angular momentum of the inflowing gas can be neglected, i.e. to the case that the specific angular momentum of the gas is $<< R_{core}v_{core}$ **(see Bonnell & Bate 2005 for a discussion of the accretion of angular momentum in the case of a turbulent medium). We here restrict ourselves to the case of inflow of gas with zero angular momentum in order to make a comparison with the (non-rotating) simulations of Bonnell & Bate 2002: evidently further simulations will be required to test the validity of our analysis in the case of more complex velocity fields for the gas.** If, in addition, the core remains in a state of approximate virial equilibrium (which is again likely in the adiabatic growth regime), then $v_{core}^2 \propto M_{core}/R_{core}$. Thus, the core mass and radius (M_{core} and R_{core}) are related via the adiabatic relationship:

$$R_{core} \propto M_{core}^{-3} \tag{3}$$

According to this equation, continued accretion drives the core ineluctably towards higher densities (note that this adiabatic scaling implies that the mass density in the core scales as M_{core}^{10}!). This runaway increase in core density would then apparently only be halted by the exhaustion of the accretion flow onto the core (either because the whole

cluster gas mass had accreted onto the core, or because the supply into the core was limited either by the effects of feedback or by consumption by star formation in the rest of the cluster).

However, this inexorable core growth requires that the core shrinkage timescale ($t_{\dot{M}}$) is less than the timescale on which the core is puffed up by discrete interactions within the core (either the creation of dynamical binaries and the associated transfer of kinetic energy into the motions of third bodies or the aggregate effect of two body encounters). **The timescale for these effects depends on the number of stars in the core: in the case of populous clusters ($N_{core} > 100$), this timescale is the conventional two body relaxation timescale (see Binney & Tremaine 1987, eqn. (4-9)), while in the case of low N_{core}, this timescale has been quantified by the simulations of Bonnell & Clarke 1999 as $\sim Nt_{dyn_{core}}$ (see also the discussion in Heggie & Hut 2003, p. 257).** [1] **We therefore write:**

$$t_{puff} \sim fN_{core}t_{dyn_{core}} \tag{4}$$

where $f = 1$ for $N_{core} < 100$ and $f -> 0.1/\ln N_{core}$ as $N_{core} -> \infty$. We thus propose that the core can continue to shrink in response to adiabatic accretion provided that

$$\frac{M_{core}}{M_{clus}}t_{dyn} < \mathbf{f}N_{core}t_{dyn_{core}} \tag{5}$$

or

$$t_{dyn_{core}} > \frac{\bar{m}}{\mathbf{f}M_{clus}}t_{dyn} \tag{6}$$

where $\bar{m}_{core}$ is the mean stellar mass in the core. Since the dynamical timescale is simply proportional to the inverse square root of the mean enclosed density, it then follows that the core density and mean cluster density ($\bar{\rho}$) are then related via the inequality

$$\rho_{core} < \left(\frac{\mathbf{f}M_{clus}}{\bar{m}_{core}}\right)^2 \bar{\rho} \tag{7}$$

We can apply this formula to the simulations of Bonnell & Bate 2002, for which $M_{clus} \sim 10^3 M_\odot$ (since the simulation was designed to mimic the Orion Nebula Cluster) and where the mean stellar mass in the core is $\sim 3M_\odot$. We find, by inspection of Figure 2 of Bonnell & Bate, that the core density indeed increases by about five orders of magnitude compared with the mean cluster density. Bonnell & Bate found that further shrinkage of the core was at this stage offset by puffing up due to few body interactions in the dynamically decoupled cluster core.

[1] Note that we are assuming here that this puffing up is not offset by the outward transference of energy into orbital motions of stars in the surrounding cluster, i.e. we assume that the Nbody evolution of the core is dynamically decoupled from that of the surrounding cluster. Given the runaway growth in core density, it is probably reasonable to assume such dynamical decoupling. Were this not the case, the core could be driven to higher densities than we estimate here

3 THE MEAN NUMBER OF COLLISIONS PER STAR.

The number of collisions experienced by a star is given by the product of the collision rate at the highest density achieved by the stellar core and the lifetime of the core in that highest density state. We have argued above that this latter is given by the timescale on which the core puffs up by few-body effects which, in the case that the velocity dispersion in the core is v_{core}, is approximately equal to the timescale on which a star experiences a gravitationally focused encounter with another star, i.e. an encounter with a star within impact parameter:

$$r_{enc} = \frac{G\bar{m}}{v_{core}^2}. \tag{8}$$

Since stellar collisions result from situations where the closest approach distance r_{coll} is generally $<< r_{enc}$, and since all encounters within r_{enc} are, by definition, in the gravitationally focused regime, we can write the fraction of gravitationally focused encounters that lead to collision as the ratio of r_{coll} to r_{enc}. The mean number of collisions per star is thus given by

$$\bar{n}_{coll} = \frac{r_{coll}}{r_{enc}} \tag{9}$$

where we here define r_{coll} as the separation of closest approach which would lead, directly or indirectly, to a stellar collision. The indirect route to stellar collisions involves the interaction between a passing star and a hard binary such that, during the interaction, two of the stars pass within a stellar radius of each other. From the three-body computations of Davies, Benz & Hills (1994), one may deduce that in the case of gravitationally focused encounters for which the intruder's nominal closest approach to the binary (if the binary were treated as a point mass) was of order the binary separation - the minimum encounter distance is commonly about a tenth of the binary separation. Hence we would expect that stellar collisions could result from such encounters in the case of binaries with separation of about 10 stellar radii (or about 1 A.U.). Thus provided massive stars in the cluster core are frequently located in binaries of this separation (consistent with the observed binary properties of OB stars in general: see Mason et al 1998, Garcia & Mermillliod 2001) then r_{coll} may exceed r_* (the stellar radius) by an order of magnitude. If we define v_{coll} as the escape velocity at separation r_{coll} (rather than r_*), we can then rewrite (9) as:

$$\bar{n}_{coll} = \left(\frac{v_{core}}{v_{coll}}\right)^2 \tag{10}$$

In order to be useful, we need to relate v_{core}, the velocity dispersion in the core at its maximum density, to the velocity dispersion of the parent cluster, $\tilde{v}$. A condition of approximate virial equilibrium implies that velocity dispersion scales with system mass and density as $\propto M^{1/3}\rho^{1/6}$, so that, at maximum density, equation (7) implies that

$$\frac{v_{core}}{\tilde{v}} \sim \left(\mathbf{f}N_{core}\right)^{1/3} \tag{11}$$

Thus the mean number of collisions per star in the core is given by:

$$\bar{n}_{coll} \sim \left(\mathbf{f} N_{core}\right)^{2/3}\left(\frac{\tilde{v}}{v_{coll}}\right)^2 \quad (12)$$

Likewise, the total number of collisions expected in the core is:

$$\bar{N}_{coll} \sim \mathbf{f^{2/3} N_{core}^{5/3}}\left(\frac{\tilde{\mathbf{v}}}{\mathbf{v_{coll}}}\right)^{\mathbf{2}} \quad (13)$$

These equations allow one to assess which clusters are suitable for producing accretion induced stellar collisions. If we invoke a high binary fraction, so that $r_{coll} \sim 10 r_*$, then $(\tilde{v}/v_{coll})^2$ is typically $\sim 10^{-3} - 10^{-2}$ for clusters with $\tilde{v}$ in the range $5-20$ km s^{-1}. This places a lower limit of $N_{core} \sim 20-100$ before one expects collisions to occur, and a lower limit of $N_{core} \sim 10^3 - 3\times10^4$ in order for the majority of stars in the core to be involved in collisions.

We thus see that - given that v_{coll} is just set by the stellar mass-radius relationship and the binary fraction among massive stars - the main *environmental* determinants of whether stellar collisions are expected are the velocity dispersion, $\tilde{v}$, and the membership number, N_{core} of the adiabatically contracting core. This raises the obvious question of how the value of N_{core} relates to the parameters of the larger system. Clearly, once a sub-system starts to contract adiabatically, its subsequent evolution becomes increasingly adiabatic. Simple application of the adiabatic condition, suggests that a sub-system will embark on the path of adiabatic contraction if its internal velocity dispersion exceeds that of the parent cluster (gas reservoir), although this hypothesis has not been subject to numerical investigation. In this case, the existence and scale of any adiabatic core would depend on the details of the cluster's radial density profile. We note that our simple one zone model for the evolution of the adiabatic core does not describe the possibility that the number of stars involved in adiabatic evolution may increase during the collapse. Evidently, this is an issue that requires numerical investigation. It is thus difficult, pending further simulations, to decide how N_{core} relates to the mass of the parent cluster, apart from the obvious expectation that the total number of stars in the cluster N_* should comfortably exceed N_{core}.

4 CONCLUSIONS

We have set up a simple theoretical model which demonstrates why current simulations of accretion of gas onto the cores of stellar clusters do not drive stellar densities to the point where stellar collisions occcur. This is because the conditions modeled - motivated by the properties of the Orion Nebula Cluster - involve free fall velocities of a few km s $^{-1}$ and - with a total cluster mass of around $10^3 M_\odot$ - the development of an adiabatic core containing only a few tens of stars. According to equation (13), the expected number of collisions in such a system is ~ 0.01. With such a low free fall velocity the core's shrinkage timescale is relatively long, while, with a core containing such a low number of stars, the core's dissolution timescale due to Nbody effects is relatively short. Consequently, the system cannot shrink to the densities required for stellar collisions.

We find however that the number of collisions is strongly dependent on both the free fall velocity of the parent cluster ($\tilde{v}$) and the number of stars in the adiabatic core (N_{core}), scaling as $N_{core}^{5/3}\tilde{v}^2$. Hence, even without increasing the number of stars in the core, an increase in $\tilde{v}$ to ~ 20 km s^{-1} increases the expected number of collisions to of order unity. Since it is also likely that N_{core} would be larger for a more massive over-all system, it would appear that stellar collisions are a likely outcome in more populous, more tightly bound clusters such as globular clusters or super star clusters.

We note that the scenario outlined in this paper bears comparison with another scenario that leads to stellar collisions in an ultra-dense cluster core, namely the action of the Spitzer (mass segregation) instability, (see Portegies Zwart & McMillan 2002, Gürkan et al 2004, Portegies Zwart et al 2004 and Freitag et al 2006a),b)). In both cases, the onset of stellar collisions raises the possibility of producing supermassive stars and stellar remnants, the latter being discussed as possible seed black holes for eventual merger into supermassive black holes in galactic nuclei.

In the case of the Spitzer instability, high densities are achieved by two body relaxation in the presence of a stellar mass spectrum and the limiting factor is whether the timescale for this process is sufficiently short so as to allow collisions to occur over the (few Myr) timescale on which the most massive (largest cross-section) stars expire as a supernova. In the present case, the timescale for core shrinkage is of order the cluster dynamical timescale ($\sim 10^5$ years) and therefore collapse is obviously fast enough compared with stellar evolutionary timescales. Here, instead, the issue hinges on the maximum density that is achievable and whether adiabatic shrinkage is offset by either two body effects (as discussed here) or by the quenching of accretion into the cluster core. Although detailed consideration of this latter effect is beyond the scope of this paper, we here note that the efficacy of feedback by ionising radiation is highly (and negatively) dependent on gas density (Dale et al 2005). This factor thus reinforces the fact that effective shrinkage of the cluster core is favoured in the densest clusters.

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